ED-616

M.A./M.Sc. 3rd Semester
Examination, March-April 2021

MATHEMATICS

Optional (C)

Paper - III

Fuzzy Sets and its Applications - I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Let $A_i \in F(x)$ for all $i \in I$, where I is an index set. Then prove that

$$\bigcup_{i \in I} {}^{\alpha -} A_i \leq {}^{\alpha}\left(\bigcup_{i \in I} A_i \right)$$

(*b*) State and prove second decomposition theorem.

(*c*) Prove that the standard Fuzzy intersection is the only idempotent *t*-norm.

2. (*a*) Prove that a *t*-norm *i* and an involute fuzzy complement *C*, the binary operation *u* on [0, 1] defined by $u(a, b) = c\ (i\,(a), c\,(b))$ for all $a,b \in [0, 1]$ is a *t*-conorm such that <*i*, *u*, *c*> is a dual triple.

(*b*) Let *A* and *B* are Fuzzy numbers with triangular shape in a Fuzzy equation as

$$A = \begin{cases} 0 & \text{for } x \leq 3, x > 5 \\ x-3 & \text{for } 3 < x \leq 4 \\ 5-x & \text{for } 4 < x \leq 5 \end{cases}$$

$$B = \begin{cases} 0 & \text{for } x \leq 12, x > 32 \\ (x-12)/8 & \text{for } 12 < x \leq 20 \\ (32-x)/12 & \text{for } 20 < x \leq 32 \end{cases}$$

Find the solution of equation $A \cdot X = B$.

(*c*) Prove that that $< i, u, c >$ be a dual triple. Then prove that the Fuzzy operations i, u, c satisfy the law of excluded middle and the law of contradiction.

3. (*a*) Prove that for Fuzzy sets

$$MIN\ [A,\ MAX\ (A\ B)] = A.$$

(*b*) Let R be a reflexible Fuzzy relation on X^2, where $|X| = n \geq 2$. Then prove that $R_{T(i)} = R^{(n-1)}$.

(*c*) Solve the following Fuzzy relation equation using max-min composition

$$P \circ \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = \begin{bmatrix} .6 & .6 & .5 \end{bmatrix}$$

4. (*a*) If R symmetric, then prove that each power of R is symmetric.

(*b*) Explain Fuzzy compatibility relations.

(*c*) Explain Fuzzy graphs.

5. (*a*) Prove that every possibility measure 'Pos' on a finite power set $P(x)$ is uniquely determined by a possibility distributive function $r : X \to [0, 1]$ via the formula. $\text{pos}(A) = \max r(x)$ for each $A \in P(X)$.

(*b*) Explain the Evidence theory.

(*c*) Let a given finite body of evidence (ε. m) be nested, then prove that for all A, $B \in P(X)$, we have

(*i*) $\text{bel}(A \cap B) = \min[\text{bel}(A), \text{bel}(B)]$

(*ii*) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$

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