ED-311

M.A./M.Sc 1st Semester
Examination, March-April 2021

MATHEMATICS

Paper - III

Topology - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Prove that $2^{\aleph_0} = c$, where card $N = \aleph_0$ and card $R = c$.

(*b*) Let X be a non-empty set and $\tau = \{G \subset X \mid G^c \text{ is countable}\} \cup \{\phi\}$. Then prove that (X, τ) is a topological space.

(*c*) Let A be a subset of a topological space X. Then prove that A is closed iff $D(A) \subset A$, where $D(A)$ is the derived set of A.

Unit-II

2. (*a*) Prove that a subset of a topological space is open iff it is a neighbourhood of each of its points.

(*b*) Prove that a mapping $f: X \to Y$, where X and Y are topological spaces, is continuous iff the inverse image of every member of a base for Y is open in X.

(*c*) Prove that second countability is a hereditary property.

Unit-III

3. (*a*) Prove that limits of sequences are unique in a Hausdorff space.

(*b*) Let $f: X \to [0, 1]$ be continuous. For each $t \in R$ let $F_t \{ x \in X : f(x) < t\}$. Then prove that the indexed family $\{F_t : t \in R\}$ has the following properties :

(*i*) F_t is an open subset of X for each $t \in R$;

(*ii*) $F_t = \phi$ for $t < 0$;

(*iii*) $F_t = X$ for $t > 1$;

(*iv*) For any s, $t \in R$, $s < t \Rightarrow \overline{F}_s \subset F_t$.

(*c*) Prove that regularity is a hereditary property.

Unit-IV

4. (*a*) Prove that a topological space X is compact iff it has a subbase ρ with the property that every cover of X by members of ρ has a finite subcover.

(*b*) Prove that a first countable, countably compact space is sequentially compact.

(*c*) Let (X, τ) be a topological space, $X^* = X \cup \{\infty\}$ and $\tau^* = \tau \cup \{G \subset X^* | \infty \in G$ and $X^* - G$ is compact and τ- closed subset of $X\}$. Prove that (X^*, τ^*) is a compact topological space.

Unit-V

5. (*a*) Prove that a metric space is compact iff it is countably compact.

(*b*) Prove that a topological space X is disconnected iff there exists a non-empty proper subset of X which is both open and closed in X.

(*c*) Prove that the components of a totally disconnected space (X, τ) are the singleton subsets of X.