ED-1010

PGDCA 2nd Semester
Examination, May-June 2021

PGDCA - 107

Database Management System

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) What do you understand by data, information and knowledge ? Also explain the concept of DBMS.

(*b*) Explain various types of database languages in detail.

(*c*) Write short notes on any **two** of the following :

(*i*) Database Administration roles

(*ii*) Importance of data dictionary

(*iii*) Relational data model

2. (*a*) What do you mean by ER-model? Explain entity attributes and relationships with example.

(*b*) Draw an E-R diagram forn library management system.

(*c*) Explain the following with example:

(*i*) Primary key

(*ii*) Candidate key

(*iii*) Super key

(*iv*) Foriegn key

(*v*) Unique key

3. (*a*) Explain relational algebra and its concept with example.

(*b*) Explain the concept of the domain relational calculus.

(*c*) Explain simple and complex queries using relational algebra.

4. (*a*) What is decomposition? Explain functional dependencies with example.

(*b*) What do you mean by normalization? Explain 1NF, 2NF and 3NF with suitable example.

(*c*) Differentiate between BCNF and 3NF.

5. (*a*) Explain DDL, DCL and DML with suitable example.

(*b*) Explain management of roles and granting roles and privilege in DBMS.

(*c*) Explain the following with example :

(*i*) Order by clause

(*ii*) Join clause

(*iii*) Group function

(*iv*) Where clause

ED-463

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - V

Advanced Discrete Mathematics-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define connectivity of a graph and prove that if the intersection of two paths in a graph is a disconnected graph then the union of the two paths has at least one circuit.

(*b*) Define Tree and prove that a graph is a tree if and only if there is one and only path between every pair of vertices.

(*c*) Define planar graph and state and prove Euler's formula for connected planar graph.

Unit-II

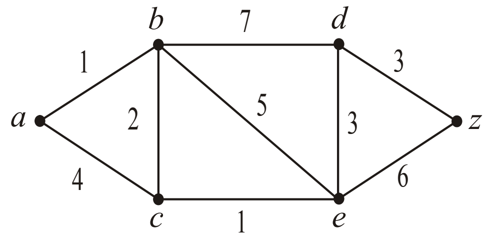
2. (*a*) Define fundamental cut sets and prove that every circuit has an even number of edges in common with every cut set.

(*b*) Explain the incidence matrix and adjacency matrix of a graph.

(*c*) The necessary and sufficient condition for a connected graph G to be an Euler graph is that 'all vertices of G are of even degree'. Show that.

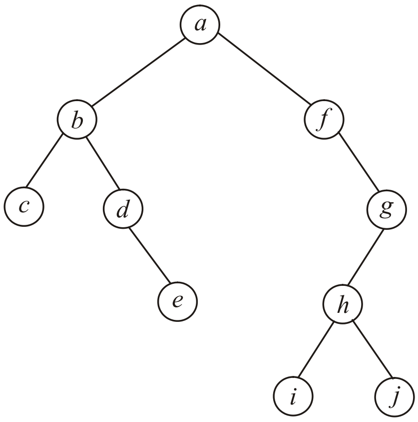
Unit-III

3. (*a*) Define weighted graph and write an algorithm for shortest path in weighted graph and use it to find shortest path from a to z in the graph shown in fig. where numbers associated with the edges are the weights.



(*b*) Explain Warshall's algorithm and lct $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3)$ $(3, 4)$ $(2, 1)\}$ be a relation on R then find transitive closure of R.

(*c*) Explain Tree Traversals and determine the order in which the vertices of the binary tree given below will be visited under (*i*) In order (*ii*) Pre order (*iii*) Post order



Unit-IV

4. (*a*) Design a finite state machine *M* which can add two binary numbers and compute the sum of 101110 and 010011.

(*b*) Define equivalent states and find π_0, π_1 and π_2 for the following finite state machines :

	State	Input		Output
		0	1	
⇒	S_0	S_1	S_5	0
	S_1	S_0	S_5	0
	S_2	S_6	S_0	0
	S_3	S_7	S_1	0
	S_4	S_0	S_6	0
	S_5	S_7	S_2	1
	S_6	S_0	S_3	1
	S_7	S_0	S_2	1

(*c*) Define homomorphism. Let S be any state in a finite state machine and let x and y be any words then $f(S, xy) = f(f(S, x), y)$ and $g(S, xy) = g(f(S, x), y)$.

Unit-V

5. (*a*) Define finite state automaton and design a finite state automaton that accepts those strings over $\{0, 1\}$ such that the number of zeros is divisible by 3.

(*b*) Construct deterministic finite state automaton equivalent to the following non deterministic finite state automaton $M = (\{0, 1\}, \{S_0, S_1\}, S_0, \{S_1\}, f\}$ where f is given by the table

S \ I	f: 0	f: 1
S_0	$\{S_0, S_1\}$	$\{S_1\}$
S_1	ϕ	$\{S_0, S_1\}$

(*c*) Write any two differences between Moore and Mealy Machine and consider the Mealy Machine described by the transition tables. Construct a Moore Machine which is equivalent to the Mealy Machine.

	Present state	Input $a = 0$		Input $a = 1$	
		state	output	state	output
$\Rightarrow$	S_1	S_3	0	S_2	0
	S_2	S_1	1	S_4	0
	S_3	S_2	1	S_1	1
	S_4	S_4	1	S_3	0

ED-462

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - IV

Advanced Complex Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Weierstrass factorization theorem.

(*b*) To prove for $\operatorname{Re} z > 1$,

$$G(z)\overline{|(z)} = \int_0^{\infty}\left(e^t - 1\right)^{-1} t^{z-1} dt$$

(*c*) State and prove Mittag-Leffler's theorem.

Unit-II

2. (*a*) State and prove Schwarz reflection principle.

(*b*) Show that the series $\sum_{n=0}^{\infty} \frac{z^n}{z^{n+1}}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(z-i)^{n+1}}$ are analytic continuation to each other.

(*c*) Let $\gamma : [0, 1] \to C$ be a path and let $\{(f_t, D_t) : 0 \le t \le 1\}$ be an analytic continuation along γ. For $0 \le t \le 1$ let $R(t)$ be the radius of convergence of the power series expansion of f_t about $z = \gamma(t)$. Then either $R(t) = \infty$ or $R : [0, 1] \to (0, \infty)$ is continuous.

Unit-III

3. (*a*) State and prove Harnack's theorem.

(*b*) Let G be a region and $f : \delta_{\infty} G \to R$ a continuous function. Then show that $u(z) = \sup \{\phi(z) : \phi \in P(f, G)\}$ defines a harmonic function u in G.

(c) Let G be a region, then prove that :

(i) The matrix space $H(G)$ is complete.

(ii) If $\{u_n\}$ is a sequence in $H(G)$ such that $u_1 \le u_2 \le$ ..... then either $u_n(z) \to \infty$ uniformly on compact subset of G or $\{u_n\}$ converges in $H(G)$ to a harmonic function.

Unit-IV

4. (a) Define order of an entire function. Find the order of polynomial $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n,\ a_n \ne 0$.

(b) State and prove Poisson-Jensen formula.

(c) State and prove Borel's theorem.

Unit-V

5. (a) State and prove Bloch's theorem.

(b) State and prove Schottky's theorem.

(c) State and prove $\frac{1}{4}$-theorem.

ED-460

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - II

Real Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Let f is bounded on $[a, b]$, f has only finitely many points of discontinuity on $[a, b]$ and α is continuous at every point at which f is discontinuous. Then show that $f \in \mathbb{R}(\alpha)$.

(*b*) Let α be a monotonically increasing function on $[a, b]$ and $\alpha' \in \mathbb{R}\,[a, b]$. Let f is bounded real function on $[a, b]$. Then prove that $f \in \mathbb{R}(\alpha)$ if and only if $f\alpha' \in \mathbb{R}\,[a, b]$. In that case

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\,dx.$$

(*c*) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\Lambda_\gamma = \int_a^b |\gamma'(t)|\,dt$.

Unit-II

2. (*a*) Prove that a countable union of measurable set is a measurable set.

(*b*) Show that a function is simple iff it is measurable and assumes only a finite number of values.

(*c*) Let $\{E_i\}$ be an infinite decreasing sequence of measurable sets; that is, a sequence with $E_{i+1} \subset E_i$, for each $i \in N$. Let $m(E_i) < \infty$ for at least one $i \in N$. Then show that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$.

Unit-III

3. (*a*) The class $\mathscr{B}$ of all μ^*-measurable sets is a σ-algebra of subsets of X. If $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$, then prove that $\bar{\mu}$ is a complete measure on $\mathscr{B}$.

(*b*) Show that the set function μ^* is an outer measure.

(*c*) Let (X, S, μ) be a σ-finite measure space, Σ a semi ring of sets such that $S \subset \Sigma \subset \mathscr{B}$, and $\bar{\mu}$ is a measure on Σ. If $\bar{\mu} = \mu$ on S, then prove that If $\bar{\mu} = \mu^*$ on Σ.

Unit-IV

4. (*a*) Evaluate the four derivatives at $x = 0$ of the function given by

$$f(x) = \begin{cases} ax.\sin^2\left(\frac{1}{2}\right) + bx.\cos^2\left[\frac{1}{x}\right] & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ a'x.\sin^2\left(\frac{1}{x}\right) + b'x.\cos^2\left[\frac{1}{x}\right] & \text{if } x < 0 \end{cases}$$

where $a < b$ and $a' < b'$.

(*b*) State and prove Vitali's covering theorem.

(*c*) Let f be a bounded and measurable function defined on $[a, b]$ if

$$F(x) = \int_a^x f(t)\,dt + F(a)$$

then $F'(x) = f(x)$ *a.e.* in $[a, b]$.

Unit-V

5. (*a*) Prove that the $\underline{|p}$ spaces are complete.

(*b*) State and prove Riesz theorem.

(*c*) State and prove Egoroff's theorem.

ED-459

M.A./M.Sc. 2nd Semester
Examination, May-June 2021

MATHEMATICS

Paper - I

Advanced Abstract Algebra-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Show that Rc is large for $c \neq 0$, $c \in R$ and R is a noetherian integral domain.

(*b*) Show that for any noetherian ring R each ideal contains a finite product of prime ideals.

(*c*) Can we prove every submodule of a noetherian module is finitely generated ? How ?

Unit-II

2. (*a*) Prove that regular elements in $A(V)$ form a group.

(*b*) In V define T by

$$\left(\sum_{n=0}^{3} \alpha_n x^n\right) T = \sum_{n=0}^{3} \alpha_n (x+1)^n .$$

Compute the matrix of T in the basis $(1, 1+x, 1+x^2, 1+x^3)$.

(*c*) If V is an n-dimensional vector space over F, then for given $T \in A(V)$ there exists a non-trivial polynomial $g(x) \in F[x]$ of degree at most n^2, such that $g(T) = 0$. Prove it.

Unit-III

3. (*a*) Show that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

(*b*) Find all possible rational canonical forms and elementary divisors for the 6×6 matrix in F_6 having $(x-1)(x^2+1)^2$ as minimal polynomial.

(*c*) Define nilpotent transformation and show that *ST*-*TS* is nilpotent iff $S, T \in A_F(V)$, *ST*-*TS* commutes with *S* and *F* is of characteristics zero.

Unit-IV

4. (*a*) Obtain the Smith normal form and rank for

$$\begin{bmatrix} -(x+3) & 2 & 0 \\ 1 & -x & 1 \\ 1 & -3 & -(x+2) \end{bmatrix}.$$

(*b*) Show that if *V* is a finite dimensional vector space over *F*, then *V* is a finitely generated $F[x]$ module.

(*c*) Let $T \in \mathrm{Hom}_F(V, V)$. Then show that there exists a basis of *V* with respect to which the matrix of *T* is $A = \mathrm{diag}(B_1, B_2, ...Br)$ where B_i is the companion matrix of a certain unique polynomial $f_i(x)$, $i = 1, 2, ... r$ such that $f_1(x) \mid f_2(x) \mid ..... \mid f_r(x)$.

Unit-V

5. (*a*) Find the rational canonical form of a matrix whose invariant factors are $x+2$, $x^2 - x - 6$, $x^3 - 2x^2 - 5x + 6$.

(*b*) Find Jordan canonical form of a matrix with characteristics polynomial $p(x) = (x-1)^2 (x+1)$.

(*c*) Find invariant factors, elementary divisors and Jordan canonical form of the matrix

$$\begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}.$$

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ED-770

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Optional - A

Paper - V

Programming in ‘C’ (with ANSI Features) - II

Time : Three Hours] [*Maximum Marks* : 70

Note : Answer any **two** parts from each question. All questions carry equal marks.

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1. (*a*) What is static storage class ? Explain with suitable example.

(*b*) Demonstrate local and global variable using suitable example.

(*c*) Explain ANSI rules for the syntax and semantics of the storage class.

2. (*a*) What is Pointer ? Explain pointer arithmetics with example.

(*b*) How to pass Array as argument in a function ? Explain with suitable example.

(*c*) Demonstrate pointer to pointer with suitable example.

3. (*a*) What is recursive function ? Explain with example.

(*b*) Demonstrate macro substitution with suitable example.

(*c*) What is conditional compilation ? Explain with suitable example.

4. (*a*) Write a program to input Roll No, Name and Marks of any three subjects then calculate total marks and percentage using structure.

(*b*) Write a program to demonstrate, how the memory is allocated dynamically and release.

(*c*) Write a program to add a new node in single link list and display the value of all nodes.

5. (*a*) What is error ? How to handle errors in 'C' languages ?

(*b*) Demonstrate the reading and writting in a file with suitable example.

(*c*) Explain any five input/output functions in 'C' language with example.

ED-768

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Optional - A

Paper - IV

Operations Research

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Use dynamic programming to solve

Minimize $z = p_1 \log p_1 + p_2 \log p_2 \; ..... + p_n \log p_n$

Subject to the constraints :

$p_1 + p_2 + p_3 + ... + p_n = 1$ and $p_j \geq 0 \; (j=1, 2, ... n)$

(*b*) What is principle of optimality ? Write the recursive equation approach to solve dynamic programming problem.

(*c*) Use dynamic programming to solve the following L.P.P.

Maximize $z = 3x_1 + 5x_2$

Subject to the constraints :

$x_1 \le 4,$

$x_2 \le 6,$

$3x_1 + 2x_2 \le 18$ and

$x_1, x_2 \ge 0$

Unit-II

2. (*a*) Consider a 'modified' form of 'matching biased wins' game problem. The matching player is paid ₹ 8 if the two coins turns both heads and ₹ 1 if the coins turns both tails. The non-matching player is paid ₹ 3 when two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy ?

(*b*) Solve the following problem graphically :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \end{array}$$

(*c*) For the following playoff matrix, find the value of the game and the strategies of player A and B by using Linear Programming :

$$\begin{array}{c} \\ \text{Player } A \end{array} \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \text{Player } B \\ \begin{bmatrix} 3 & -1 & 4 \\ 6 & 7 & -2 \end{bmatrix} \end{array}$$

Unit-III

3. (*a*) Solve the following integer P.P. :

Maximize $z = 2x_1 + 3x_2$

Subject to the constraints :

$-3x_1 + 7x_2 \leq 14,$

$7x_1 - 3x_2 \leq 14,$

$x_1, \; x_2 \geq 0$

and are integers

(*b*) Use branch and bound method to solve the following L.P.P. :

Minimize $z = 4x_1 + 3x_2$

Subject to the constraints :

$5x_1 + 3x_2 \geq 30,$

$x_1 \leq 4,$

$x_2 \leq 6,$

$x_1, \; x_2 \geq 0$

and are integers

(*c*) Maximize $z = x_1 + x_2$

Subject to the constraints :

$3x_1 + 2x_2 \leq 5,$

$x_2 \leq 2,$

$x_1, \; x_2 \geq 0$ and

x_1 is an integer.

Unit-IV

4. (*a*) Write the applications of operations reserach to industrial problems.

(*b*) Explain petroleum and refinery operations.

(*c*) Explain blending problems.

Unit-V

5. (*a*) Obtain the necessary and sufficient conditions for the optimum solutions of the following NLPP :

Minimize $z = f(x_1, x_2)$

$= 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraints :

$x_1 + x_2 = 7$ and

$x_1, x_2 \geq 0$

(*b*) Use Wolfe's method to solve

Max. $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to the constraints :

$x_1 + 2x_2 \leq 2$ and

$x_1, x_2 \geq 0$

(*c*) Solve the following quadratic programming problems by using Beale's method :

Maximize $z = 2x_1 + 3x_2 - x_1^2$

Subject to the constraints :

$x_1 + 2x_2 \leq 4$ and

$x_1, x_2 \geq 0$

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ED-766

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - III (C)

Fuzzy Set Theory and Its Applications-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define fuzzy propositions with properties and examples.

(*b*) Define fuzzy quantifiers with examples.

(*c*) Write the method of inference from conditional and qualified propositions.

Unit-II

2. (*a*) Let f be a function defined by $f(a) = e^a$ for all $a \in [0, 1]$. Find the fuzzy intersection, fuzzy union, fuzzy implication and fuzzy compliment generated by f.

(*b*) Explain approximate reasoning and fuzzy language with one such example.

(*c*) Write the interpolation method and show that $B_2^1 \subseteq B_4^1 \subseteq B_1^1 = B_3^1$.

Unit-III

3. (*a*) Write a short note on design of fuzzy controllers.

(*b*) Discuss possible ways of fuzzyfying the general dynamic system.

(*c*) Discuss the design of a air conditioner fuzzy controller.

Unit-IV

4. (*a*) Define defuzzification and write any two methods of defuzzification.

(*b*) Aggregate graphically the fuzzy sets :

$$A_1 = \frac{0}{0}, \frac{.3}{1}, \frac{.3}{2}, \frac{.3}{3}, \frac{.3}{4}, \frac{0}{5}$$

$$A_2 = \frac{0}{3}, \frac{.5}{4}, \frac{.5}{5}, \frac{.5}{6}, \frac{0}{7}$$

$$A_3 = \frac{0}{5}, \frac{1}{6}, \frac{1}{7}, \frac{0}{8}$$

and solve it by the centroid method.

(*c*) Solve the following fuzzy linear programming problems

Max. $z = 6x_1 + 5x_2$

Subject to

$(5, 3, 2)x_1 + (6, 4, 2)x_2 \leq (25, 6, 9)$

$(5, 2, 3)x_1 + (2, 1.5, 1)x_2 \leq (13, 7, 4)$

$x_1, x_2 > 0.$

Unit-V

5. (*a*) Let each individual of four decision makers has a total preference ordering $P_i\ (i \in N)$ on a set of alternatives $X = \{a, b, c, d\}$ as

$P_1 = (a, b, d, c)$; $P_2 = (a, c, b, d)$;

$P_3 = (b, a, c, d)$; $P_4 = (a, d, b, c)$

Find the fuzzy preference relation. Also find α-cuts of the fuzzy relation and group level of agreement concernng the social choice denoted by the total ordering (a, b, c, d).

(*b*) Explain individual and multiperson decision making in fuzzy environment.

(*c*) Explain construction of an ordering of all given alternatives by Shimura method.

ED-763

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - II

Partial Differential Equations and Mechanics

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Hamilton ODE.

(*b*) Derive Hopf-Lax formula.

(*c*) For asymptotics in L_{∞} norm, there exists a constant C such that $|u(x,t)| \le C/\sqrt{t}$.

Unit-II

2. (*a*) Use separation of variables to solve the porous medium equation

$$u_t = \Delta\left(u^{\Upsilon}\right) = 0 \text{ in } \mathbb{R}^n \times (0, \infty).$$

(*b*) State and prove Plancherel's theorem.

(*c*) Derive Hopf-Cole transformation.

Unit-III

3. (*a*) Explain about vanishing viscosity method for Burger's equation.

(*b*) Write about asymptotics for linear terms.

(*c*) Define Majorants. Show that if $f = \sum_{\alpha} f_{\alpha} \cdot x^{\alpha}$ converges for $|x| < r$ and $0 < s\sqrt{n} < r$ then f has a majorant for $|x| < s\sqrt{n}$.

Unit-IV

4. (*a*) State and prove the principle of least action.

(*b*) Explain about Poincare-Cartan integral.

(*c*) Show that the transformation

$$p = q \cot p, \; Q = \log\left(\frac{1}{q}\sin p\right)$$

is cannonical.

Unit-V

5. (*a*) State and prove the relation between Lagrange's and Poisson's brackets.

(*b*) Prove that the Poisson bracket of two constants of motion is itself a constant of the motion.

(*c*) State and prove Jacobi Identity through Poisson bracket.

ED-762

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - I

Functional Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove closed graph theorem.

(*b*) Let X be a Banach space and Y be a normed linear space. Let $\{T_i\}$ be a non-empty set of continuous linear transformation from X into Y, such that $\{T_i(x)\}$ is bounded for each x and X, then show that is $\{\|T_i\|\}$ is bounded.

(*c*) Let T be a bounded linear transformation from a Banach space X into a normed linear space Y. Then show that the openness of T implies the completness of Y.

Unit-II

2. (*a*) Let X and Y be normed linear space. Then show that $B(X, Y)$ the set of all bounded linear transformations from X into Y, is a normed linear space.

(*b*) Let X is a Banach space. Then show that X is reflexive if and only if X^* is reflexive, where X^* is the conjugate space of a normed linear space X.

(*c*) Let E be a real normed linear space and let M be a linear subspace of E. If $f \in M^*$, then show that there is a $g \in E^*$ such that $f \subset g$ and $\| g \| = \| f \|$.

Unit-III

3. (*a*) State and prove Bessel's inequality.

(*b*) If X is an inner product space and $x, y \in X$, then show that $|(x, y)| \leq \|x\| \, \|y\|$.

(*c*) Show that a Banach space is a Hilbert space if and only if the parallelogram law holds.

Unit-IV

4. (*a*) State and prove Riesz Representation theorem.

(*b*) Prove that every Hilbert space is reflexive.

(*c*) Let T be an operator on a Hilbert space H. Then there exists a unique operator T^* on H such that

$$(Tx, y) = (x, T^*y)$$

for all $x,\ y \in H$.

Unit-V

5. (*a*) If T_1 and T_2 are self-adjoint, then show that $T_1 T_2$ is self-adjoint if and only if they commute, i.e. $T_1 T_2 = T_2 T_1$.

(*b*) State and prove generalized Lax-Milgram theorem.

(*c*) If T is a normal operator on a Hilbert space H and D is any scalar, then show that $T\text{-}\lambda\, I$ is also normal.